PHYSICS 152A

I am not Prof. Butov

There will be no lecture Thur. Jan 8

Next lecture is Tue, Jan 13 (next week)

Books mentioned in the lecture:

"Solid State Physics"
by Neil W. Ashcroft, N. David Mermin

and

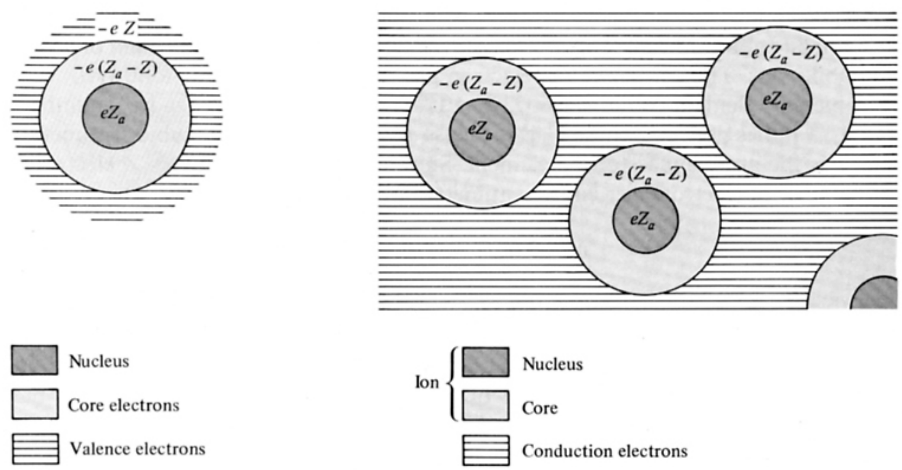
"Condensed Matter Physics"
by Michael P. Marder
(includes more modern developments)

The free electron theory of metals
The Drude theory of metals

Paul Drude (1900): theory of electrical and thermal conduction in a metal
application of the kinetic theory of gases to a metal,
which is considered as a gas of electrons

mobile negatively charged electrons are confined in a
metal by attraction to immobile positively charged ions

isolated atom



in a metal

nucleus charge eZ_a
Z <u>valence electrons</u> are weakly bound to the nucleus (participate in chemical reactions)
$Z_a - Z$ <u>core electrons</u> are tightly bound to the nucleus (play much less of a role in chemical reactions)

in a metal – the <u>core electrons</u> remain bound to the nucleus to form the metallic ion
the <u>valence electrons</u> wander far away from their parent atoms
called <u>conduction electrons</u> or <u>electrons</u>

density of conduction electrons in metals ~$10^{22} - 10^{23}$ cm^{-3}

r_s – measure of electronic density

r_s is radius of a sphere whose volume is equal to the volume per electron

$$\frac{4\pi r_s^3}{3} = \frac{V}{N} = \frac{1}{n} \qquad r_s = \left(\frac{3}{4\pi n}\right)^{1/3} \sim \frac{1}{n^{1/3}}$$

mean inter-electron spacing

in metals $r_s \sim 1 - 3$ Å (1 Å$= 10^{-8}$ cm) $\qquad r_s/a_0 \sim 2 - 6$

$$a_0 = \frac{\hbar^2}{me^2} = 0.529\text{Å} \text{ – Bohr radius}$$

- electron densities are thousands times greater than those of a gas at normal conditions
- there are strong electron-electron and electron-ion electromagnetic interactions

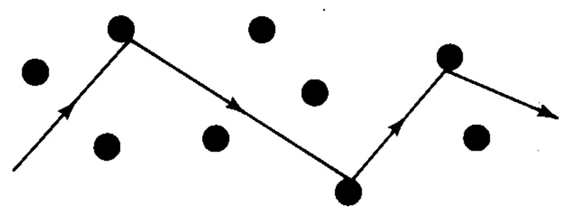
↓

in spite of this the Drude theory treats the electron gas
by the methods of the kinetic theory of a neutral dilute gas

The basic assumptions of the Drude model

1. between collisions the interaction of a given electron
with the other electrons is neglected ← independent electron approximation
and with the ions is neglected ← free electron approximation

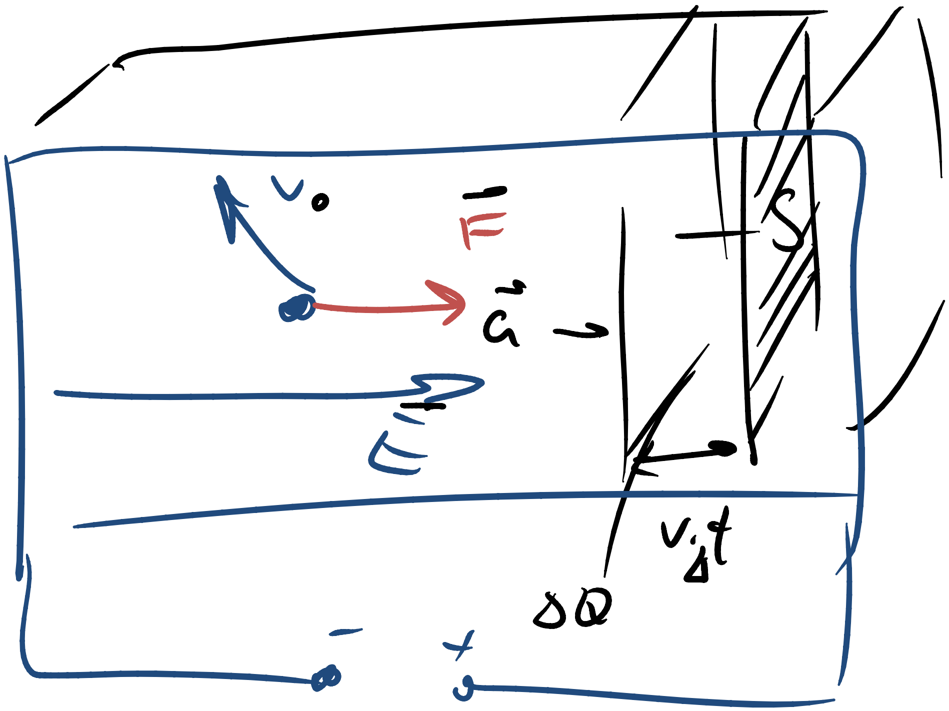
2. collisions are instantaneous events
Drude considered electron scattering off
the impenetrable ion cores



the specific mechanism of the electron scattering is not considered below

3. an electron experiences a collision with a probability per unit time $1/\tau$
dt/τ – probability to undergo a collision within small time dt
randomly picked electron travels for a time τ before the next collision
τ is known as the <u>relaxation time</u>, the <u>collision time</u>, or the <u>mean free time</u>
τ is independent of an electron position and velocity

4. after each collision an electron emerges with a velocity that is randomly directed and
with a speed appropriate to the local temperature



$$I = e \cdot S \langle V \rangle \cdot n_{cond}$$

$$\Delta Q = S \cdot v \cdot \Delta t \cdot e \cdot n_{cond}$$

$$\frac{\Delta Q}{\Delta t} = I = S \cdot v \cdot e \cdot n_{cond}$$

$$I = S \cdot \frac{eE}{m} \cdot \tau \cdot e \cdot n_{cond}$$

$$I = U \cdot S \cdot \frac{e^2}{L} \cdot \frac{\tau}{m} \cdot n_{cond}$$

$$E = \frac{U}{\ell}$$

$$\vec{F} = e \cdot \vec{E}$$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{e\vec{E}}{m}$$

$$\vec{v}(t) = \vec{v}_0 + \vec{a} \cdot t$$

$$\langle \vec{v}(t) \rangle = \underbrace{\langle \vec{v}_0 \rangle}_{0} + a \langle t \rangle$$

$$\langle v \rangle = a \cdot \tau = \frac{eE}{m} \cdot \tau$$

DC electrical conductivity of a metal

$V = RI$ Ohm's low

the Drude model provides an estimate for the resistance

introduce characteristics of the metal which are independent on the shape of the wire

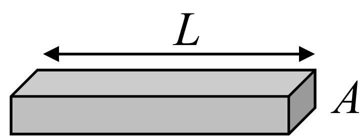
$$\mathbf{E} = \rho \mathbf{j} \qquad \mathbf{j} = \sigma \mathbf{E}$$

$j=I/A$ – the current density

ρ – the resistivity

$R=\rho L/A$ – the resistance

$\sigma = 1/\rho$ – the conductivity



$$\mathbf{j} = -en\mathbf{v}$$

v is the average electron velocity

$$\mathbf{v} = -\frac{e\mathbf{E}}{m}\tau \qquad \mathbf{j} = \left(\frac{ne^2\tau}{m}\right)\mathbf{E}$$

$$\mathbf{j} = \sigma \mathbf{E} \qquad \sigma = \frac{ne^2\tau}{m}$$

$$\tau = \frac{m}{\rho n e^2}$$

at room temperatures
resistivities of metals are typically of the order of microohm centimeters (μohm-cm)
and τ is typically $10^{-14} - 10^{-15}$ s

$$\frac{1}{2} m v^2 = \frac{3}{2} k T$$

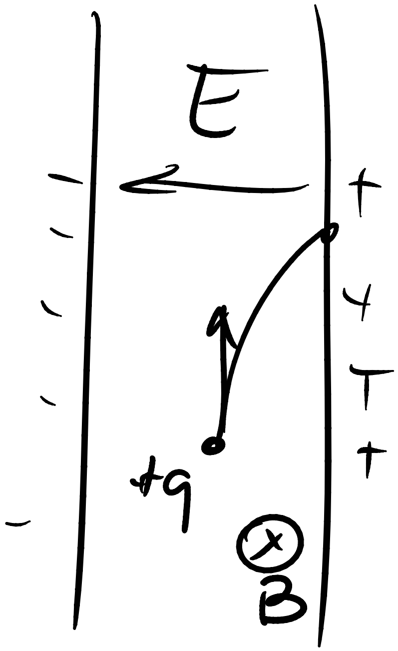
<u>mean free path</u> $l = v_0 \tau$
v_0 – the average electron speed
l measures the average distance an electron travels between collisions
estimate for v_0 at Drude's time $1/2\, m v_0^2 = 3/2\, k_B T \rightarrow$ $v_0 \sim 10^7$ cm/s $\rightarrow$ $l \sim 1 - 10$ Å
consistent with Drude's view that collisions are due to electron bumping into ions

at low temperatures very long mean free path can be achieved
$l > 1$ cm $\sim 10^8$ interatomic spacings!
the electrons do not simply bump off the ions!

the Drude model can be applied where
a precise understanding of the scattering mechanism is not required

$\downarrow$

particular cases: electric conductivity in spatially uniform static magnetic field
and in spatially uniform time-dependent electric field



$$E \cdot q = v \cdot q \cdot B$$

$$U = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + n \cdot \hbar\omega_c$$

motion under the influence of the force $\mathbf{f}(t)$ due to spatially uniform electric and/or magnetic fields

average momentum ↓ average velocity ↓

$$\mathbf{p}(t) = m\mathbf{v}(t)$$

<u>equation of motion</u>
for the momentum per electron

$$\frac{d\mathbf{p}(t)}{dt} = -\frac{\mathbf{p}(t)}{\tau} + \mathbf{f}(t)$$

electron collisions introduce a frictional damping term for the momentum per electron

Derivation:

$$\mathbf{p}(t+dt) = \underbrace{\left(1-\frac{dt}{\tau}\right)}_{\text{fraction of electrons that does not experience scattering}} \cdot \left[\mathbf{p}(t) + \mathbf{f}(t)dt\right] + \underbrace{\frac{dt}{\tau}}_{\text{scattered part}} \cdot \underbrace{0}_{\text{total loss of momentum after scattering}}$$

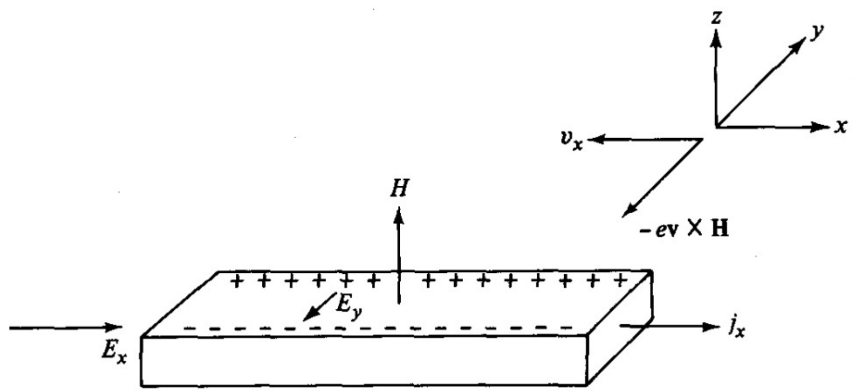
$$\mathbf{p}(t+dt) = \mathbf{p}(t) + \mathbf{f}(t)dt - \frac{\mathbf{p}(t)}{\tau}dt + O\left(dt^2\right)$$

$$\frac{\mathbf{p}(t+dt) - \mathbf{p}(t)}{dt} = \mathbf{f}(t) - \frac{\mathbf{p}(t)}{\tau} + O\left(dt\right)$$

$$\frac{d\mathbf{p}(t)}{dt} = \mathbf{f}(t) - \frac{\mathbf{p}(t)}{\tau}$$

Hall effect and magnetoresistance

Edwin Herbert Hall (1879): discovery of the Hall effect



the Hall effect is the electric field developed across two faces of a conductor in the direction **j×H** when a current **j** flows across a magnetic field **H**

the Lorentz force $\mathbf{F}_L = -\frac{e}{c}\mathbf{v}\times\mathbf{H}$

in equilibrium $j_y = 0$ → the transverse field (the Hall field) E_y due to the accumulated charges balances the Lorentz force

quantities of interest:

magnetoresistance (transverse magnetoresistance) $R(H) = R_{xx} = \frac{V_x}{I_x}$

resistivity $\rho(H) = \rho_{xx} = \frac{E_x}{j_x}$

Hall (off-diagonal) resistance $R_{yx} = \frac{V_y}{I_x}$

Hall resistivity $\rho_{yx} = \frac{E_y}{j_x}$

the Hall coefficient $R_H = \frac{E_y}{j_x H}$

R_H → measurement of the sign of the carrier charge

R_H is positive for positive charges and negative for negative charges force acting on electron $\mathbf{f} = -e\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right)$

equation of motion
for the momentum per electron $\frac{d\mathbf{p}}{dt} = -e\left(\mathbf{E} + \frac{1}{mc}\mathbf{p}\times\mathbf{H}\right) - \frac{\mathbf{p}}{\tau}$

in the steady state p_x and p_y satisfy

$$0 = -eE_x - \omega_c p_y - \frac{p_x}{\tau}$$

$$0 = -eE_y + \omega_c p_x - \frac{p_y}{\tau}$$

$$\omega_c = \frac{eH}{mc}$$

← cyclotron frequency
frequency of revolution of a free electron in the magnetic field H

$$m\omega_c^{\ 2} r = \frac{e}{c}(\omega_c r)H$$

$$\nu_c = \frac{\omega_c}{2\pi} \sim 1GHz$$

at H = 0.1 T

multiply by $-ne\tau/m$

the Drude model DC conductivity at H=0

$$j = -ne\frac{p}{m}$$

$$\sigma_0 = \frac{ne^2\tau}{m}$$

→

$$\sigma_0 E_x = \omega_c \tau j_y + j_x$$

$$\sigma_0 E_y = -\omega_c \tau j_x + j_y$$

$j_y = 0$ →

$$E_y = -\left(\frac{\omega_c \tau}{\sigma_0}\right) j_x = -\left(\frac{H}{nec}\right) j_x$$

$j_x = \sigma_0 E_x$ the resistance does not depend on H

$$R_H = -\frac{1}{nec}$$

$R_H \rightarrow$ measurement of the density

$\omega_c\tau << 1$ weak magnetic fields – electrons can complete only a small part of revolution between collisions

$\omega_c\tau >> 1$ strong magnetic fields – electrons can complete many revolutions between collisions measurable quantity – <u>Hall resistance</u> $\rho_H = R_H H$

$$\rho_H = \frac{V_y}{I_x} = -\frac{H}{n_{2D}ec}$$

for 3D systems $n_{2D} = nL_z$

for 2D systems $n_{2D} = n$

$\mathbf{E} = \rho\mathbf{j}$ $\mathbf{j} = \sigma\mathbf{E}$

in the presence of magnetic field the resistivity and conductivity becomes tensors

for 2D: $\rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}$ → $\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$

$$\sigma_0 E_x = \omega_c \tau j_y + j_x$$

$$\sigma_0 E_y = -\omega_c \tau j_x + j_y$$

↓

$$E_x = \frac{1}{\sigma_0} j_x + \frac{\omega_c \tau}{\sigma_0} j_y$$

$$E_y = -\frac{\omega_c \tau}{\sigma_0} j_x + \frac{1}{\sigma_0} j_y$$

→ $\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 1/\sigma_0 & \omega_c\tau/\sigma_0 \\ -\omega_c\tau/\sigma_0 & 1/\sigma_0 \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$

$$\rho = \begin{pmatrix} 1/\sigma_0 & \omega_c\tau/\sigma_0 \\ -\omega_c\tau/\sigma_0 & 1/\sigma_0 \end{pmatrix}$$

→

$$\rho_{xx} = \frac{1}{\sigma_0} = \frac{m}{ne^2\tau}$$

$$\rho_{xy} = \frac{\omega_c\tau}{\sigma_0} = \frac{H}{nec}$$

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

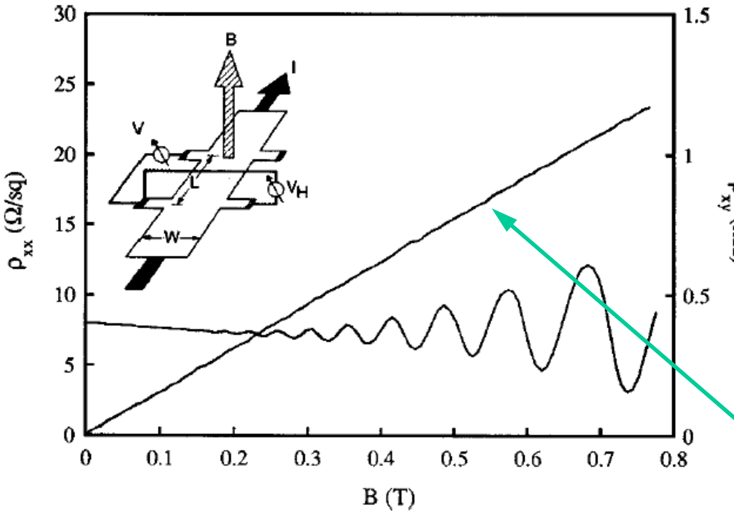
$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}^{-1}$$

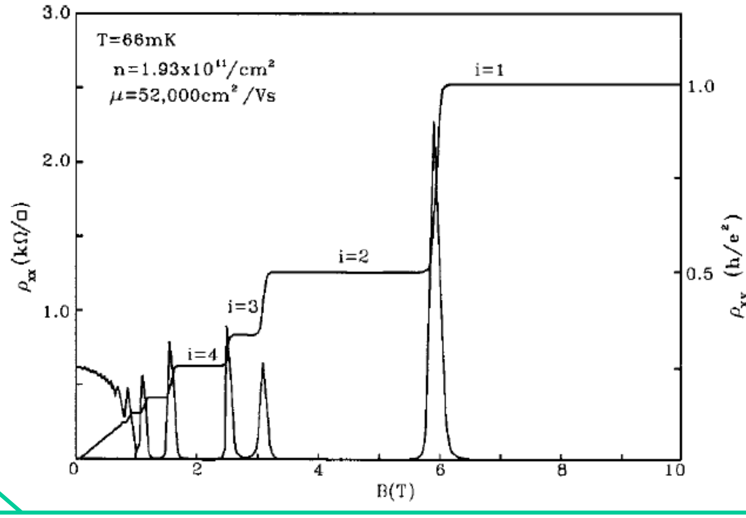
$$\sigma_{xx} = \sigma_{yy} = \frac{\sigma_0}{1+(\omega_c\tau)^2}$$

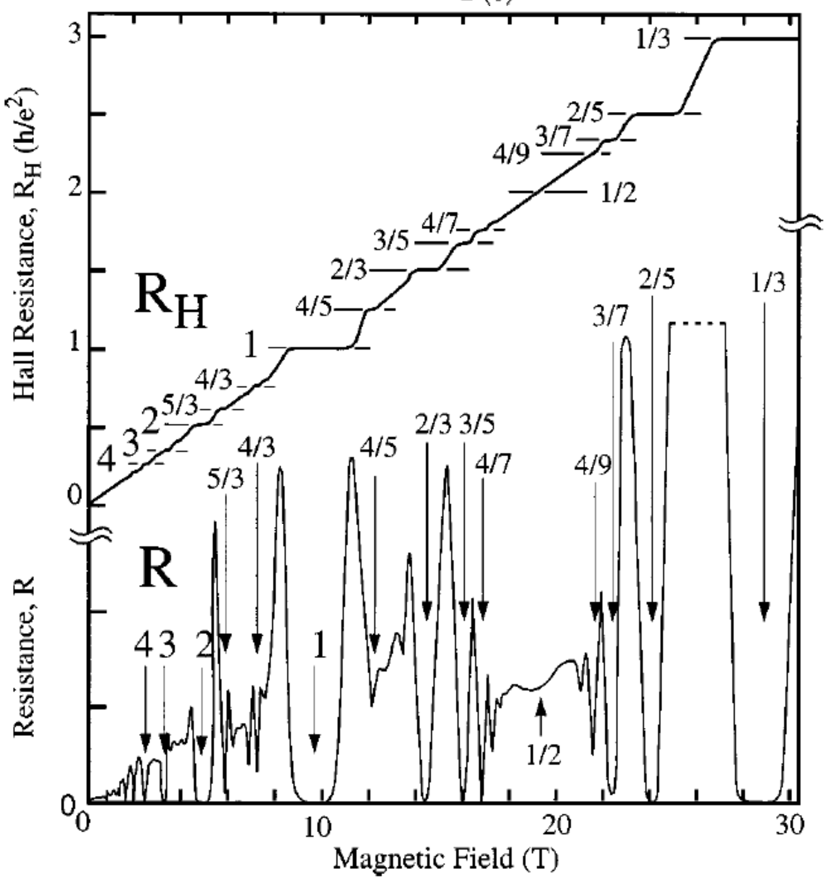
$$\sigma_{xy} = -\sigma_{yx} = \frac{-\sigma_0\omega_c\tau}{1+(\omega_c\tau)^2}$$

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}$$

$$\sigma_{xy} = -\frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}$$







from D.C. Tsui, RMP (1999) and from H.L. Stormer, RMP (1999)

Hall resistance $\rho_H = R_H H = -\frac{H}{nec}$

weak magnetic fields $\omega_c\tau \ll 1$

$$\rho_{xx} = \frac{1}{\sigma_0} = \frac{m}{ne^2\tau}$$ the Drude model

$$\rho_{xy} = \frac{\omega_c\tau}{\sigma_0} = \frac{H}{nec}$$ the classical Hall effect

strong magnetic fields $\omega_c\tau \gg 1$

quantization of Hall resistance $\rho_{xy} = \frac{h}{\nu e^2}$

at integer and fractional $\nu = n\Big/\left(\frac{eH}{hc}\right)$

the integer quantum Hall effect

and the fractional quantum Hall effect